

## STABILITY OF CHOICES AMONG UNCERTAIN ALTERNATIVES

By WILLIAM H. MCGLOTHLIN, The Rand Corporation

Recently there has been an increasing interest in the theory of games and decision making, with the development of various models and strategies for determining choices. When decisions are made among alternative offers whose outcomes are unknown at the time of the choice, as in a gambling game, three main variables are involved. These are: (1) the amount wagered or risked,  $x$ ; (2) the size or value of the prize,  $y$ ; and (3) the objective probability of a successful outcome,  $P$ . If it is hypothesized that the individual's best strategy is to maximize the expected value,  $E$ , of his choices, it becomes a simple matter to predict uncertain decisions. Expectation,  $E$ , is defined as the summation of the products of all possible outcomes and the probability attached to each. Losses are treated as negative outcomes.

Human behavior often does not follow the above strategy, however, as in the case of buying insurance, or in accepting a gamble in which the expectation is negative. It may be assumed that the individual reacts to the psychological counterparts of  $x$ ,  $y$ , and  $P$ ; these are: utility of bet,  $U(x)$ ; utility of prize,  $U(y)$ ; and subjective probability,  $P'$ . Edwards and others have hypothesized that choices between uncertain alternatives can be predicted on the basis of maximization of 'subjectively expected utility,'  $SEU$ .<sup>1</sup>

$$SEU = \sum P'_i U_i$$

where  $U_i$  represents the utility of the  $i$ th possible outcome of the bet and  $P'_i$  represents the subjective probability of the outcome.

There have been some experimental attempts to measure utilities and subjective probabilities as functions of the corresponding objective scales. The validity of these functions is often in question, for it is usually necessary to assume objective probabilities, or linear functions thereof, in order to find a subjective utility function, and to make similar assumptions about utility when deriving subjective probability. For instance, suppose that 75% of the time subjects ( $S$ s) preferred

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<sup>1</sup> Ward Edwards, Probability-preferences in gambling, this JOURNAL, 66, 1953, 349-364.

a probability of 0.2 of winning \$4.00 to a probability of 0.4 of winning \$2.00. From these results we could conclude one of the following: (1) psychologically, an objective probability of 0.4 is less than twice the objective probability of 0.2; (2) the utility of \$4.00 is more than twice the utility of \$2.00; (3) neither subjective probability nor the utility of money, as valued by  $S$ , can be expressed on the corresponding objective scales.

Instead of presenting  $S$  with alternatives yielding equal expected values as in the above example, we may design the experiment such that choices are made between events yielding unequal expected values. By using some sort of competitive bidding, or by means of an information processing center such as a pari-mutuel betting machine, it is possible to have  $S$  himself establish the expected values of various combinations of probabilities and prizes. Negative expected values for a particular bet, probability, and prize would indicate at least one of the following: (1)  $P' > P$ ; (2)  $U(\$x) < x$  utiles;  $U(\$y) > y$  utiles; where utiles refers to the unit of measurement for utility. Positive expected values for a given set of values would indicate the reverse relationship; *i.e.* (1)  $P' < P$ ; (2)  $U(\$x) > x$  utiles; (3)  $U(\$y) < y$  utiles.<sup>2</sup>

### PROBLEM

The present study assumes there is sufficient comparability among individuals to make an investigation of group risk-taking behavior meaningful. It is a statistical study of 9605 thoroughbred horse-races, mostly from California tracks, during the years 1947-1953. The primary purpose is to examine the stability of risk-taking behavior over a series of events. One approach to this question is to determine the expected values of constant-size wagers for a range of probabilities of success  $P$ . This yields an  $E$ -vs.- $P$  pattern and can be repeated for a series of risk-taking events, *i.e.* races. The stability of this pattern throughout the racing day allows some inferences to be made about the stability of subjective probability and utility for wager and prize over a series of events. It is also possible to obtain information about stability of risk-taking behavior that is independent of the expected-value. Variability of size of average wager over a series of events can be determined as well as preferences among wagers having equal expectations ( $E$ s) but different probabilities of success. Some limited information is available concerning differential preferences between winning and losing bettors.

### PROCEDURE

*Pari-mutuel wagering.* Betting on horse-races is quite different from most other forms of gambling. The betting population establishes the odds, or the amount of money each horse in the race will return if successful. At a race track there is a

<sup>2</sup> For a more complete discussion of the problems involved in this type of model construction, see R. M. Thrall, C. H. Coombs, and R. L. Davis (eds.), *Decision Processes*, 1954, 255-285.

totalizator board on which the current odds-to-win on each horse appear. The odds are recalculated and flashed on the board at about 45-sec. intervals, keeping the public informed as to the amount of backing each horse has received up to that time. It is not until the final bet is made that the exact odds on each horse are established.

Betting may be for place or show in addition to win. If the horse finishes first or second the place ticket is redeemable; for the show ticket, the horse may finish first, second, or third. The amount returned on these tickets is independent of the position the horse occupies at the finish.

*Win.* The winning odds posted by the track are given by the formula:  $a_i = [(1 - t) \cdot \Sigma A - A_i] / A_i$ , where  $a_i$  = odds that the  $i$ th horse will finish first;  $t$  = proportion track takes;<sup>3</sup>  $\Sigma A$  = amount bet in the win pool on all horses for the race being considered; and  $A_i$  = amount bet on the  $i$ th horse to win.

The odds found in this manner are rounded downward to the nearest multiple of 5¢ (10¢ in some states). The odd cents so deducted are called breakage. The winning ticket pays an amount that includes these odds plus the original bet.

*Place.* Place odds at the track are determined by the following formula:  $b_1 = [(1 - t) \cdot \Sigma B - (B_1 + B_2)] / 2B_1$ , where  $b_1$  = place odds for horse finishing in the first position;  $\Sigma B$  = amount bet in the place pool on all horses for the race being considered; and  $B_1, B_2$  = amount bet that the horses finishing in the first and second positions will place. The place odds for the horse finishing in the second position,  $b_2$  are found by replacing  $B_1$  with  $B_2$  in the denominator of the above formula.

*Show.* Show odds are determined as follows:  $c_1 = [(1 - t) \cdot \Sigma C - (C_1 + C_2 + C_3)] / 3C_1$ , where  $c_1$  = show odds for horse finishing in the first position;  $\Sigma C$  = amount bet in the show pool on all horses for the race being considered; and  $C_1, C_2, C_3$  = amount bet on the horses finishing in the first, second and third positions to show. The show odds for the horses finishing in the second and third positions,  $c_2$  and  $c_3$ , are found by replacing  $C_1$  with  $C_2$  and  $C_3$ , respectively, in the denominator of the above formula.

It is apparent from the last two formulas that the place and show odds are dependent not only on the amount of money bet on the individual horse in these categories, but also on the amount bet on the other horses that appear in the numerator. While winning odds are calculated and reported on all horses in a race whether they win or not, it is practical to report the place odds only for the first two horses, and the show odds for the first three.

*Range of odds.* The range of odds established on the horses in any given race depends primarily on how closely the horses are matched in ability. In a typical race of 9 or 10 horses the odds-to-win range from around 2-1 on the public favorite to around 50-1 on the horse receiving the least public backing. The place odds typically range from about 1-1 to 20-1, while the show odds range from about 0.5-1 to 6-1. Thus, in a 9 horse race there are 27 possible bets with a typical range of 0.5-1 to 50-1. In the case of win-betting, good approximations of these odds are available to the bettor at the time he makes his choice. As explained above, accurate estimates of place and show odds are not available at the time of the decision making, although it is virtually certain that the place and show odds will be con-

<sup>3</sup> Track take varies from 10 to 15% in the 23 states permitting pari-mutuel wagering on thoroughbred racing. In California the figure is 13%.

siderably lower than the winning odds appearing on the totalizator board for a given horse.

*Data.*<sup>4</sup> The data used in this study were obtained from the *Daily Racing Form Chart Book* and are described in Table I.<sup>5</sup> The main sample consists of 1156 days or 9248 races. In view of the fact that some of the most interesting results were found in the data for the eighth race, an additional sample of 357 eighth races was analyzed to increase the reliability of the results. Whenever the eighth-race data were combined with data for other races, they were given a weight of 1156/1513.

Some tracks schedule an additional race on Saturdays, giving a total of nine races.

TABLE I  
SOURCES OF DATA

Track	Years	Number of racing days
Hollywood Park, California	1947-1953	345
Santa Anita, California	1947-1953	348
Tanforan, California	1947, 1949-1951, 1953	212
Golden Gate Park, California	1947, 1949-1952	210
Bay Meadows, California	1951	41
Bay Meadows, California	1947-1950	168*
Jamaica, New York	1950	60*
Aqueduct, New York	1950	38*
Belmont Park, New York	1950	52*
Empire City, New York	1950	5*
Saratoga, New York	1950	34*
Total		1513

\* Only eighth races included.

To combine these races with the remainder of the data, the fifth race was omitted and the sixth race was used in place of the fifth-race data, and so on.

The study is of a statistical nature, and as such, lacks many of the controls found in the experimental laboratory setting. The population of bettors is not stable throughout the racing day due to late arrivals, early departures, and the fact that many bettors do not wager on every race. The amount bet by different individuals varies in an uncontrolled manner, such that persons wagering large amounts determine the size of the odds to a greater extent than do smaller bettors. Also, there is no direct way of studying differential behavior among those persons receiving reinforcement in the form of successful bets and those losing. Finally, the results found are strictly applicable only to the population from which they were derived, *i.e.* the horse-race betting public. The extent to which the results agree with other studies of this type gives some indication of their generality.

<sup>4</sup> The author wishes to express his appreciation to the public relations staff of Hollywood Park for the use of their records and office space during this study. Bill Haney, John Maluvius, James Sinnott, and Al Wesson were especially cooperative and patient.

<sup>5</sup> *The Daily Racing Form Chart Book*, Vols. 53-59, 1947-1953, Triangle Publication Inc., Los Angeles.

*Treatment of data.* The data have been handled in virtually the same manner as a similar study made by Griffith in 1948.<sup>6</sup> He used data from 1386 races and divided the horses into 11 groups according to the odds established on each horse in the pari-mutuel wagering. By checking the outcome of these races, the true or objective probability ( $P = \text{winners/entries}$ ) was found for each odds-group, and these odds were compared with the subjectively established public odds. In the present study, the total sample has been broken down into eight subsamples depending on the order of the race in the daily program. Each horse whose track odds to win (all are given as odds to one dollar) fell between 0.05 and 25.95 was placed in one of nine groups. The class intervals for the odds-groups were: 0.05–1.95; 2.00–2.95; 3.00–3.95; 4.00–4.95; 5.00–5.95; 6.00–7.95; 8.00–10.95; 11.00–15.95; and 16.00–25.95. Odds of greater than 25.95 were not recorded because the results would not have been sufficiently stable to be of use in the analysis.

The objective probability,  $P$ , that a horse in a particular odds-group will win the race is  $W/N$ , where  $W$  is the number of winning horses in the odds group, and  $N$  is the number of entries in that group. In discussing the expected value of bets for the various odds-groups, it is more convenient to use the expectation,  $E$ , found from the actual ratio of the amounts of money wagered, *i.e.* from the odds that would have prevailed had not the track take and breakage been deducted.<sup>7</sup> When odds are treated in this manner, positive, zero, and negative values of  $E$  have their conventional meaning. Expected value for a \$1 bet to win in a particular track odds group is:  $E = P \cdot a^* + (1 - P) \cdot -1$  where  $a^* = \text{mean corrected winning odds for a particular track odds group}$ .

*Reliability of the data.* The approximate standard error of  $E$  is  $\sigma_p (a^* + 1)$  where  $\sigma_p$  is the standard error of  $P$ . The standard error of  $a^*$  for a particular odds-group is so small compared to  $\sigma_p$  that it can be ignored. Because of the skewness of the sampling distribution for  $P$  at the extremes, it is usually not permissible to interpret the standard error of a proportion when  $P$  is as small as some of those appearing in this study, *i.e.* 0.05. The size of  $N$  in the present case is, however, very large (500–10,000), and under such conditions the standard error of  $P$  is applicable as a measure of reliability.

## RESULTS

*Expected values for entire sample.* In Table II and Fig. 1, the expected values are given as functions of odds for the total sample of 9248 races.<sup>8</sup>

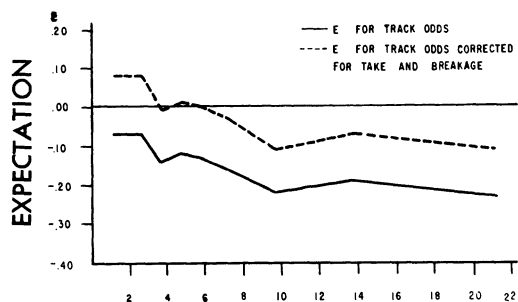
<sup>6</sup> R. M. Griffith, Odds adjustments by American horse-race bettors, this JOURNAL, 62, 1949, 290-294.

<sup>7</sup> Track odds may be corrected for track take and breakage as follows:  $a_c = [(a + 1.025)/(1 - t)] - 1$ , where  $a_c = \text{corrected odds}$ ,  $a = \text{track odds}$ , and  $t = \text{track take}$ . In the above equation 1.025 represents the original \$1.00 bet plus the correction for breakage. With the exception of Fig. 1, expected value,  $E$ , always refers to values computed from *corrected* odds. The odds-groups, however, are stated in track odds.

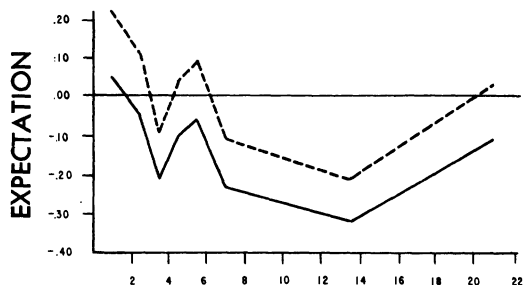
<sup>8</sup> Some examples of the raw data from which the entries in Table II were computed may be helpful to the reader. In the 1156 races which occupied the first position in the day's program, there were 771 horses whose track odds-to-win were 0.05–1 to 1.95–1. Of these, 320 won and returned an average of \$1.24 for each

TABLE II  
EXPECTED VALUES OF ONE DOLLAR BETS AS A FUNCTION OF TRACK ODDS

		Track odds								
Position of Race	Number of Races	0.05- 1.95	2.00- 2.95	3.00- 3.95	4.00- 4.95	5.00- 5.95	6.00- 7.95	8.00- 10.95	11.00- 15.95	16.00- 25.95
1	1156	.08	.04	.05	-.11	-.06	.04	-.10	.12	-.11
2	1156	.14	.13	-.05	.02	.08	-.06	-.21	-.08	-.05
3	1156	.05	.09	.06	.08	-.07	.01	.12	-.13	-.07
4	1156	.05	.10	.04	.12	-.05	-.12	-.06	-.03	-.10
5	1156	.03	.10	-.02	-.07	.02	-.07	-.02	-.02	-.14
6	1156	.11	.03	-.01	-.05	-.06	-.07	.01	-.05	-.20
7	1156	.01	.00	.00	.08	.03	.19	-.17	-.13	-.32
8	1513	.22	.11	-.09	.04	.09	-.11	-.15	-.21	.03
1-8	9248	.08	.08	-.01	.01	.00	-.03	-.11	-.07	-.11
(σ <sub>E</sub> ) <sub>1-7</sub>		.048	.064	.077	.091	.108	.086	.095	.114	.121
(σ <sub>E</sub> ) <sub>8</sub>		.053	.056	.057	.069	.089	.073	.071	.082	.121
(σ <sub>E</sub> ) Total		.017	.022	.026	.032	.038	.031	.033	.039	.044



ODDS, RACES 1-8 (N = 9248)



ODDS, EIGHTH RACES (N = 1513)

FIG. 1. EXPECTED VALUES OF ONE DOLLAR BETS AS A FUNCTION OF ODDS.

dollar bet, in addition to the original wager. The mean odds  $\delta$  ( $a^*$ ) that would have obtained had the track take and breakage not been deducted are:  $[(1.24 + 1.025)/0.87] - 1 = 1.60$ . From this,  $E = 0.415(1.60) + 0.585(-1) = 0.08$ . In the total sample of 9248 races over all eight positions in the racing program, there were 8781 horses entered at odds of 8.00-1 to 10.95-1. Of these, 661 won at an average odds of 9.32-1. The expected value is then  $-0.11$ .

A horizontal line is drawn on the graph at an expectation,  $E$ , of zero. Odds below 3-1 show an  $E$  of 0.08, odds of 3-1 to 6-1 have an  $E$  approximately equal to zero, and odds greater than 8-1 have an  $E$  of about -0.10. The  $E$  of 0.08 exceeds 0.00 by four standard errors; the difference between the obtained  $E$  of -0.10 and that of 0.00 is significant beyond the 5% level of confidence. The indifference point, where the graph of  $E$  is equal to 0.00, is located between odds of 3.5-1 and 5.5-1, or at the probability value of 0.15 to 0.22. This agrees well with Griffith's findings of 0.16 and 0.18 in two similar analyses.<sup>9</sup>

*Expected values for subsamples.* The primary purpose of this study was to investigate what change, if any, took place in the expected values-vs.-odds relationship as the racing day proceeded. To investigate any trend that might exist, the 9248 races were broken down into eight samples of 1156 races, depending on the position the race occupied in the daily program. Table II gives the expectations ( $E$ ) for each of the eight races, and standard errors for  $E$  of each odds-group. In general, the pattern of positive  $E$  for low-odds horses and negative  $E$  for the higher odds holds for the subsamples, as it does for the total sample. The first six races all yield  $E$ -vs.-odds patterns that do not differ from the pattern for the total sample by more than the sampling error.

The group of seventh races exhibits several interesting features. First, the  $E$  for odds of less than 3-1, which has been positive for all the other subsamples, is found to be roughly equal to zero. Secondly, the  $E$  for odds of 7-1 is much larger (0.19) than usual, being significantly greater (beyond the 5% level of confidence) than the corresponding value for the total sample. The third feature is the very low  $E$  (-0.32) for odds of around 21-1. This is the lowest  $E$  for any of the samples. While this pattern is quite different from those of the remainder of the data, the differences are probably not related to betting behavior on previous races. Neither of the adjacent races (sixth and eighth) shows a similar pattern. The explanation of the  $E$ -vs.-odds pattern for the seventh race apparently lies in its uniqueness as the feature race of the day. The relatively low  $E$  for the low odds may be due to the increased familiarity of the public with two or three of the favorite horses in this race. These horses usually have impressive records and are highly publicized in the local newspapers. Other horses are seldom mentioned outside of charts showing entries and results.

The group of eighth races gives the most interesting results of all the subsamples. The graph of  $E$ -vs.-odds for the eighth races, in the lower half of Fig. 1, shows two significant features. First, and most outstanding, is the  $E$  of 0.22 for odds below 2-1. This is significantly above 0.00 beyond the 1/10% level of confidence and above the corresponding value for the first seven races (0.07) beyond the 2% level. The second feature of the eighth-race graph is the sharp dip in expectation,  $E$ , at odds of 3.5-1. The  $E$  is -0.09 compared to 0.01 for the first seven races, being not quite significant at the 5% level of confidence.

In view of the relatively high  $E$  for odds of below 2-1 in the eighth races, an effort was made to investigate further this odds-group. As was explained earlier,

<sup>9</sup> Griffith, *op. cit.*, 290-294.

it was not possible to obtain the odds to place and show established on each horse as was the case in the win category. We may, however, categorize the data on the basis of *winning* odds as before, and then list the number of horses that finished first or second, *i.e.* placed, and the number that finished third or better, *i.e.* showed. The place and show odds are available for these horses and can be tabulated as before. The *E* of a place wager on horses whose odds-to-win were below 2-1 was found to be 0.24, or 6.5 standard errors above 0.00.

Unlike the seventh race, the eighth race is not unique in type. It is almost always very similar in make-up to three or four of the earlier races. The fact that the *E*-vs.-odds graph for the eighth race is quite different from that for the first seven races must be explained by a change in betting behavior, and this change is due to the position of the race in the daily program rather than the composition of the race. Horses with a high probability of winning, but with accompanying low pay-offs, become even more unpopular with the bettors in the last race.

*Amount wagered per person as a function of position in racing meet.* During the 1953 Hollywood Park season the average amount bet per person during a racing day was \$72.70. The average amount paid to the track in the form of mutuel take then was \$9.46 per person exclusive of breakage. These figures represent the mean amount bet. Since the distribution of bets is positively skewed, due to a few large bets, the median is undoubtedly lower than the mean. Probably the former is around \$50.00 bet and \$6.50 lost. There was a tendency for the amount wagered per person to increase slightly as the seasonal meet proceeded. The average amount bet per person per day for the first 10 days was \$67.10 compared to \$75.50 for the last 10 days. The weekday average was \$76.60 per person as compared to \$65.10 per person for Saturdays and holidays.

*Relation between amount bet on a race and its position in the daily program.* During the racing day the total amount wagered per race ordinarily increases for each succeeding event up to the eighth race. There is usually a slight decline in the total mutuel handle from the seventh to the eighth race. For the 1953 Hollywood Park data the increase from the first to the seventh race is fairly regular, with the amount bet in the latter race being about 1.8 times the amount wagered on the former. Some of this change is due to late arrivals and early departures, but the increase in sizes of wagers is clearly much more than can be accounted for by fluctuations in attendance.

*Probability-preferences.* Recently, Edwards, has reported several well-designed experiments using college Ss and dealing with probability-preferences in gambling.<sup>10</sup> These studies have held constant the *E* of bets and determined the preference for different probabilities by means of paired comparisons. Eight bets on a rigged pin-ball machine were used with probability values of 1/8, 2/8, . . . 8/8. In general, the results of these experiments have shown a definite preference for bets involving the probability of success 4/8, and a definite avoidance of the value 6/8. He found that these preferences were still distinguishable in experiments involving unequal *E* even though such choices violated the maximization-of-expected-value hypothesis.<sup>11</sup>

<sup>10</sup> Edwards, *op. cit.*, 349-364.

<sup>11</sup> Edwards, Probability-preferences among bets with differing expected values, this JOURNAL, 67, 1954, 56-67.



Furthermore, he found that the above probability-preference remained constant for different levels of expected values, thus demonstrating that the preferences exist independently of the attached utility variable.<sup>12</sup> Finally, Edwards carried out an experiment designed to study 'variance preferences' in gambling.<sup>13</sup> A conservative individual, wishing to minimize the variability of his assets over a series of risk-taking events should choose wagers with small amounts bet and high probability of success. Less conservative individuals may increase the variability of their assets, *i.e.* gamble on a large win at the expense of risking a large loss, by choosing to wager large amounts at low-probability values. Edwards created special situations in which the best strategy for winning bettors consisted of minimizing the variability of their assets, and the best for losing bettors consisted of maximizing the variability of their assets. The results showed that the same preferences for probabilities that had been found in earlier experiments was still the most important factor in predicting choices. Winning bettors did not change their preferences for low cost or high cost bets as the series of choices proceeded, although it would have been in their interest to do so. Losing bettors did tend to choose high-variability bets when good strategy indicated it; however, choosing bets with high variability was of less importance than the preference for a given range of probability.

The present study presents a measure of preferences for certain probabilities and asset variability in horse-race betting. In pari-mutuel wagering, the bettor may choose among win, place, and show bets. The three pools are independent, such that the amount wagered on a horse in one category has no effect on the odds on the same horse in the other two categories. The amount deducted by the track (13%) is the same for all three pools, although the factor of breakage takes a slightly larger amount from the place and show pools, since there is a higher proportion of redeemable tickets in these categories. As mentioned earlier, the typical ranges of win, place, and show odds are around 2-1 to 50-1, 1-1 to 20-1, and 0.5-1 to 6-1 respectively. Thus, the proportion of the total amount of money wagered in each pool gives a measure of preference for probability ranges among alternatives with roughly equal expected values. This is fairly analogous to Edward's measure of preference for particular probabilities. It should be noted that during a series of races these proportions may change for the group without necessarily effecting a change in the pattern of extended values discussed earlier. Fig. 2 gives the proportion of the total amount bet in the win, place, and show categories as a function of the position of the race in the daily program. The graph for the win pool shows an almost linear increase from 0.49 in the first race to 0.60 in the eighth and last race. The proportions bet in the place and show categories show corresponding decreases.

*Risk-taking events and their effect on subsequent betting.* While the group of bettors as a whole is always losing money due to the track-take, a proportion of them is winning on any given day. The question of differences in behavior between winners and losers has been raised. A partial answer may be obtained by determining what, if any, relationship exists between the odds that the winning horse pays and the amount of money wagered per person in the following race. If each person is assumed to bet the same amount, the proportion of winning bettors would be:

<sup>12</sup> Edwards, The reliability of probability preferences, this JOURNAL, 67, 1954, 68-95.

<sup>13</sup> Edwards, Variance preferences in gambling, this JOURNAL, 67, 1954, 441-452.

$Q = 0.87/(a_1 + 1.025)$ , where  $Q$  = the proportion of those persons purchasing win tickets who realize a return; and  $a_1$  = odds-to-win for horse finishing in the first position. Place and show odds were not taken into consideration. Using this measure of the proportion of the population holding successful win tickets, we tested the correlation between these values and the amount bet per person in the following race. The data from the 50-day Hollywood Park racing season were used. Saturdays and holidays were eliminated because it has been shown that these days have a smaller amount bet per person than for weekdays. This left a total of 40 racing days

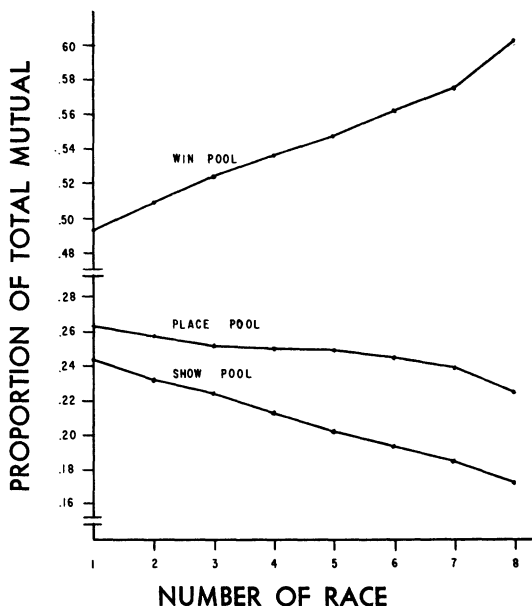


FIG. 2. PROPORTION OF TOTAL MUTUEL BET IN WIN, PLACE, AND SHOW POOLS AS A FUNCTION OF NUMBER OF RACE (Data from the 50-day meet at Hollywood Park, 1953.)

and, since there were eight races a day, seven pairs of variables to be correlated. Before computing correlation coefficients, it was necessary to correct the amounts bet per person in each race for the variance contributed by the position of the racing day in the season, since this variable increases as the season proceeds. The seven coefficients ranged from  $-0.10$  to  $-0.47$ , with one being significant beyond the 1% level of confidence, and one beyond the 5% level. Since all seven coefficients were negative, they give rather conclusive evidence that bettors increase the amount wagered more after having lost than following a successful bet.

#### DISCUSSION

*Stability of the E-vs.-odds pattern.* The general pattern is one of positive expectation,  $E$ , for low-odds wagers, high probability) and negative  $E$  for

high odds (low probability), with zero  $E$  from 3.5–1 to 5.5–1 odds ( $P = 0.15$  to  $0.22$ ). This pattern appears to have considerable stability inasmuch as it was found with minor variations for the first six subsamples of races. Marked variations of the  $E$ -vs.-odds relationship in the seventh race appear to be related to its uniqueness as the feature race. The sharp increase in the  $E$  for odds below 2–1 for the eighth race is probably due to certain segments of the population making decisions in accordance with their total financial losses for the day. Bettors apparently refrain from making bets which would not recoup their losses if successful. The increased popularity of odds of around 3.5–1 in the eighth race (indicated by the relatively low  $E$  of  $-0.09$ ) may be due to the fact that a considerable proportion of the population has lost about three times the amount they propose to wager on the last race; however, no evidence was gathered to substantiate this speculation.

In so far as we may generalize to other populations, it appears that subjects can be expected to accept low expected values when low probability-high prize combinations are involved, while demanding higher expected values in the case of high probability-low prize combinations. The central tendency-like effect shows considerable stability over a series of risk-taking events. These findings are consistent with those of Preston and Baratta, who conducted a laboratory experiment on this problem.<sup>14</sup>

*Variability of preferences in betting.* The betting behavior of the group is such as to increase the variability of their individual assets in an almost linear fashion as the racing day proceeds. This is accomplished by increasing the amount bet per person and by choosing a higher proportion of win-category wagers in preference to place and show betting. This may be partly due to the loss of resources for the group as a whole due to the track take. There is some indication that losing bettors tend to increase the size of their wagers more than do winning bettors.

It is important to note the stability of the  $E$ -vs.-odds pattern during the first six races, in spite of the fact that during this same period, size of wagers and preference for low-probability bets (win betting) are steadily increasing. The lack of change in the  $E$ -vs.-odds pattern corresponding to an increase in size of wagers would appear to indicate that the utility scale for money in the range considered is virtually the same as the objective dollar scale, and the more important psychological variable is subjective probability. On the other hand, neither is the increasing popularity of win

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<sup>14</sup> M. G. Preston and Philip Baratta, An experimental study of the auction-value of an uncertain outcome, this JOURNAL, 61, 1948, 183-193.

betting (low  $P$  of success) during the first six races reflected in the  $E$ -vs.-odds pattern. If the group's subjective evaluation of low probability-high prize wagers increases over a series of risk-taking decisions, we would expect an intensification of the negative  $E$  for high-odds and positive  $E$  for low-odds pattern. This does not occur until the last race. These results suggest the inadequacy of a model for predicting risk-taking decisions based solely on the maximization of 'subjectively expected utility.' Allais has suggested that the variances involved in the wager may also be an important factor in this type of decision making.<sup>15</sup> The results of the present study tend to confirm this prediction. This is not in agreement with Edwards' laboratory findings which indicated that variance-preferences were of minor importance compared to probability-preferences in gambling.<sup>16</sup> Edwards also found probability-preferences to be relatively stable, which is not in agreement with the present finding. Perhaps the discrepancy is due in part to the difference between college and horse-race betting populations.

#### SUMMARY

By means of a statistical analysis of 9605 horse-races, this study sought to obtain information about the stability of decision-making behavior over a series of risk-taking events. In general, the group tended to accept probability-prize combinations whose expected values were less for low-probability wagers than for high ones. This tendency was relatively stable over a series of decisions, and was for the most part, independent of decreasing group resources, size of average wagers, and change in group probability-preferences. The group behaved in a manner such as to increase the variability of their assets as a series of risk-taking events proceeded. This was accomplished by increasing the size of the wager and choosing lower probabilities (win bets) with accompanying prospective higher returns. There was some indication that losing bettors increased the size of their wagers more than did winning bettors.

The relatively stable  $E$ -vs.-odds pattern over a series of events in which sizes of wagers and preferences for probability values show consistent changes raises some questions about decision-making models. It was shown that a model making use of subjective probability and utility functions alone does not account for the results found here. In the present study, variance-preferences also play an important role in determining choices among risky alternatives.

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<sup>15</sup> Maurice Allais, Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école américaine, *Econometrica*, 21, 1953, 503-546.

<sup>16</sup> Edwards, Variance preferences in gambling, 441-452.