The Risk of the Roll: A Mathematical Analysis of Casino Dice Games

An Honors Thesis (HONRS 499)

By

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Abstract

A mathematical and probabilistic analysis has been performed on two casino, dice games, one historic and one current. On-The-Level 36 was a popular game of the 1920s which used six dice, and craps is one of modern day's most fast-paced, most exciting, and most complicated games played in the casino. The house advantage of each game will be computed. A focus will be placed on craps due to its complexity and the number of bets available to gamblers. It also has among the best odds that a gambler is likely to see in the casino (less than 1% house advantage!). Like all games, gamblers must avoid the sucker bets and follow the optimal strategy. The probabilities and expected values behind various bets on the craps table will be discussed. I will analyze the optimal strategy and present a craps game simulator (coded in Excel VBA) which is used to analyze different strategic nuances associated with this optimal strategy.

Acknowledgments

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On-The-Level 36

History

On-The-Level 36 is an antique gambling game developed by Charles Fey and Company. There are six dice on the inside of the machine which get scrambled when the user pushes the lever. The payout corresponds to the sum of the numbers shown on the dice. What made this game so attractive to gamblers was that up to $200 could be won on a $1 bet! While this might seem like a good reason to play this game, there is much more to consider. There is also a significant amount of mystery that surrounds On-The-Level 36, its history, and its functionality.

Charles Fey is renowned in the history of gambling as the inventor of the slot machine. Fey was born in 1862 in the village of Vohringen in Bavaria, which is located in present-day Germany (Charles). Originally, his name was August Fey. It was not until later in his life when he moved to the United States that he changed his name to the more American-sounding name, Charles. At the age of 14, Fey got his first job working in a factory, and it was here that he developed a passion for mechanics. He held various manufacturing jobs throughout Europe before he could afford to make the journey to the United States. Fey arrived in San Francisco, California in 1885, and in 1895, he developed the world's first slot machine, the Liberty Bell. Due to the success of this machine, Charles Fey quite his job to focus solely on the creation of gambling machines. This was the beginning of his business, Charles Fey and Company.

In 1924, Charles Fey and Company developed a gambling machine called On-The-Level 36. Today, this machine is showcased in the Nevada State Museum located in Carson City. On-The-Level was a huge hit during the roaring twenties. This was the first dice machine from the early era that accepted coins larger than a quarter. On-The-Level 36 accepted $1 coins, half dollar coins, and/or quarters. It had one slot each for the $1 and half dollar coins, and two quarter slots. The risk-taking gamblers of the roaring twenties could put up to one coin in each slot, totaling a $2.00 bet! This was a significant amount for the early 1900's! Best case scenario, a $2 bet could yield a $400 win if a 6 or a 36 was summed across the six dice.

Mysteries

The table below contains the information from the "banner" just to the left of the machine. These values are the corresponding payouts for a $1 bet. The numbers were taken directly from the banner and simply put in numerical order.

Dice Total	Pay Out ($)
6	200
7	60
8	32
9	10
10	10
11	5
12	4
13	2
14	3
15	3
16	0
17	0
18	0
19	0
20	0
21	0
22	0
23	0
24	0
25	0
26	**????**
27	3
28	2

29	3
30	4
31	0
32	6
33	10
34	40
35	60
36	200

The first interesting thing to which attention should be turned is that the payouts are not symmetrical. If the payouts start in the order $200, $60, and $32, one might expect them to end in a symmetrical pattern, $32, $60, $200. However, if one looks at the chart, the payout for a 34 is $40 rather than $32. Upon further investigation, the payouts are not close to being symmetrical. Consider the payouts for 13 and 14. From 6 through 13, the payouts steadily decrease since the likelihood of rolling that number steadily increases. What about 14? A roll of 14 pays out more than a roll of 13. The pattern of decreasing payouts is ruined. What was the purpose of this?

The second thing to note is even more intriguing and mysterious. What is the payout for a roll of 26? The number 26 is not listed on the banner anywhere! Why was this left off? What happens if a gambler rolled a 26? If the payouts of On-The-Level followed a symmetrical pattern, the payout for a 26 could be deducted simply by looking at the corresponding payout for a roll of a 16. Unfortunately, the mystery created by the seemingly random payouts helps protect the mystery surrounding the 26.

The logistics of Charles Fey's gambling machine is another mystery that remains unsolved. Imagine that a gambler walks into a casino, more likely a saloon, and sees this peculiar dice game. Intrigued, he puts a half dollar into the machine, pushes the lever, and the dice are scrambled. If he is having a lucky day, he will win money. However, the machine does not dispense any money. Is there an attendant that has to constantly sit by the machine so as to make the appropriate payout? If not, would the gambler have to call over an attendant, show the dice as proof, and then receive his winnings? What would prevent the gambler from receiving his winnings from one attendant, and then calling the attention of a different attendant a few minutes later and receiving a second payout for the same roll of the dice?

Probability

Winning $200 on a $1 bet can make On-The-Level 36 appealing to play, but is it really a good gamble? A mathematical analysis of this game tells a different story.

(a) Dice Total	(b) Pay Out	(c) # ways to roll it	(d) Probability	(e) E(Payout)
6	200	1	0.002%	0.004287
7	60	6	0.013%	0.007716
8	32	21	0.045%	0.014403
9	10	56	0.120%	0.012003
10	10	126	0.270%	0.027006
11	5	252	0.540%	0.027006
12	4	456	0.977%	0.039095
13	2	756	1.620%	0.032407
14	3	1161	2.488%	0.074653
15	3	1666	3.571%	0.107124
16	0	2247	4.816%	0
17	0	2856	6.121%	0
18	0	3431	7.354%	0
19	0	3906	8.372%	0
20	0	4221	9.047%	0
21	0	4332	9.285%	0

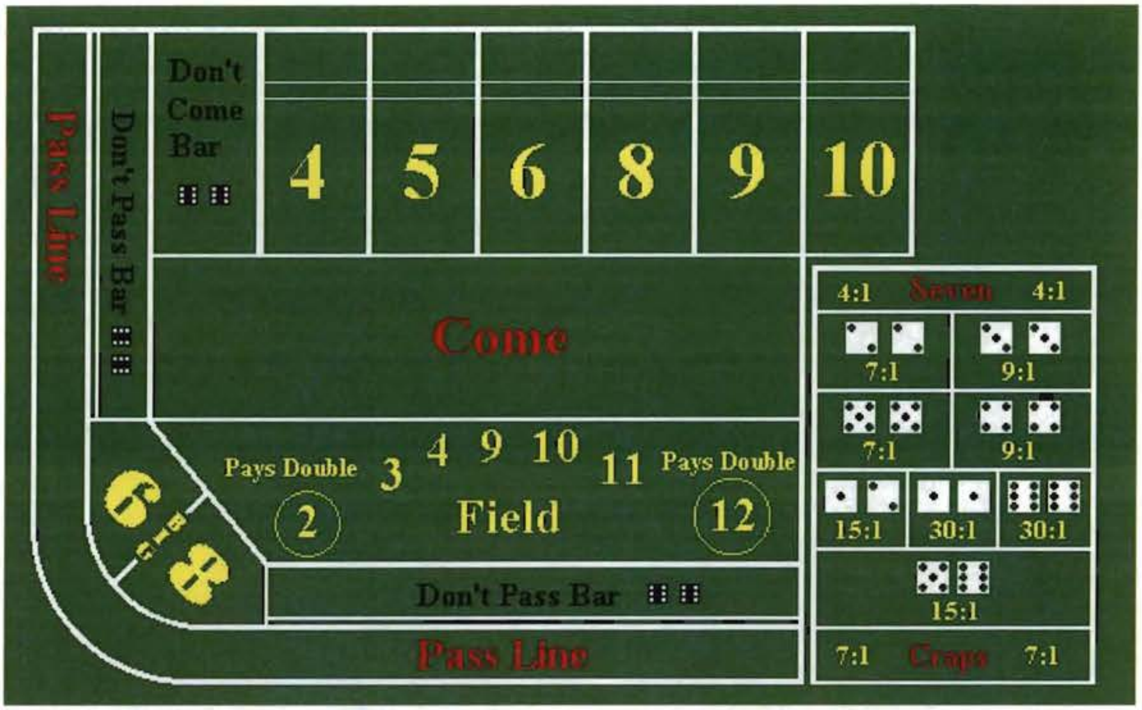
22	0	4221	9.047%	0
23	0	3906	8.372%	0
24	0	3431	7.354%	0
25	0	2856	6.121%	0
26	**????**	**2247**	**4.816%**	
27	3	1666	3.571%	0.107124
28	2	1161	2.488%	0.049769
29	3	756	1.620%	0.048611
30	4	456	0.977%	0.039095
31	0	252	0.540%	0
32	6	126	0.270%	0.016204
33	10	56	0.120%	0.012003
34	40	21	0.045%	0.018004
35	60	6	0.013%	0.007716
36	200	1	0.002%	0.004287
Total				**0.648513**

(a) All possible dice totals
(b) Given on banner
(c) Number of ways to roll that total
(d) (c) / 6^6
(e) (b) * (d)

The value of .648513 above means that according to probability, a person can expect to receive about $0.65 for every $1 that they bet. Since the payout for a roll of 26 remains a mystery, this calculation is assuming that the 26 has no payout whatsoever. If this is the case, we arrive at the 0.648513 that is above. In other words, a person can expect to lose about $0.35 for every $1 bet. These odds are terrible for gamblers! The casino has a 35% house advantage on this game! Page 15 contains a table of house advantages for some of the most popular casino games. Most house advantages tend to be within 3-7% with the worst odds being Keno at 27%. On-The-Level 36 is significantly worse than all of the other games listed in the table. While the large payout for On-The-Level 36 may seem enticing, I'd never play this game in real life. Furthermore, the probability of winning the grand prize (rolling a 6 or 36) is practically zero (at least to 4 decimal places). The house advantage for this game is simply too large for it to make mathematical sense to play. Like all games, a gambler may have a lucky day and make some money, but in the long run, On-The-Level 36 will just drain empty your wallet.

It may be interesting to reevaluate the roll of a 26. What would the payout for a 26 have to be to make this a fair game? My initial guess was that it would have to be something large like $20 or $25 to make up for the excessively large 35% house advantage. In reality, it would only have to be about $7.30. According to the table above, there is a 4.82% chance of rolling a 26. .0482*7.30 = .352. This .352 is in effect the 35% house advantage that we saw above. Even a small payout for a roll of a 26 has the possibility of drastically reducing the house advantage. For example, a payout of $2 would drop the house advantage to about 25%. As we will see in the mathematical analysis of the game of craps, a 25% house edge is unimaginably high. No matter what the procedure was for this mysterious 26, On-The-Level 36 was a game that used the unlikely possibility of winning big to entice high rollers to play a game that was stacked against them.

Craps



Introduction

Craps is a unique game for multiple reasons. It is one of the most complicated games in a casino, but as a reward to gamblers who put in the effort to learn the game, it offers among the best odds a gambler is likely to see. Especially when compared to the house edge that was calculated for On-The-Level 36, the house edge for craps will seem negligible. Additionally, craps has the potential to function almost like a team sport. Gamblers have the ability to place individual bets, but frequently, everyone at the craps table is rooting for the same number to be rolled (or for the same number NOT to be rolled). When the dice get hot and players start winning, the craps table can become one of the loudest and most energetic areas of the casino. Craps offers potential to have a lot of fun and to win big!

A Brief History of Craps

There is no definite origin of the game of craps. Its history is vague and filled with much speculation. It is believed that the game of craps has its origins in a game called hazard. There are two theories as to the history of this game and the derivation of the name "hazard." The first theory suggests that hazard was probably invented in the 11th or 12th century in England by a man named Sir William of Tyre (Craps History - Online). It was said that Sir William of Tyre and his knights would play this game for leisure when they besieged a castle called Hazarth or Asart in 1125 during the Crusades (Craps History - Online). In this case, "hazard" would be an adapted version of the name of this castle. The second theory gains its popularity from the Arabic origins of the word. Many believe that this game was invented by an Arab simply because the word "hazard" comes the Arabic word "azzah," meaning dice (The Origin). Regardless of how this game originated, it quickly became one of the most popular gambling games in England. Hazard was even mentioned by Geoffrey Chaucer in "The Canterbury Tales" (The Origin).

During the next few centuries, the game called hazard moved from England to France and across Europe. During the 17th and 18th centuries, this gambling game was one of the many items that crossed the Atlantic Ocean with the French and English immigrants. The United States, especially the southern portion (New Orleans in particular) took a profound interest in this new game. By 1813, Bernard de Mandeville of New Orleans took the game of hazard, simplified it, and transformed the game of hazard into the first version of craps. Craps is a term derived from the French word "crabs" denoting a losing dice roll of a 2 (Craps History – Craps Pit).

The game of craps was revolutionized by a man named John H. Winn. Due to his contributions to the game, Winn is renowned as the father of modern craps. He introduced the concept of the "Don't Pass Line" (Craps History – Craps Pit). This was a major advancement because for the first time, players could bet with the dice or against the dice. In effect, the "Don't Pass Line" eliminated the usefulness of fixed dice. In the original version of the game, players could only win one way, thus creating a huge incentive to fix the dice in that one way. After Winn's enhancement, a single roll of the dice could result in a profit for the 'right better' (those who bet the Pass Line) and a loss for the 'wrong better' (those who bet the Don't Pass Line) or vice versa. The added ambiguity and the additional freedom for the gambler worked together to discourage the use of fixed dice.

The astute gambler might wonder about the complementary nature of the Pass Line bet and the Don't Pass bet. If they are truly complementary, that is if the loss from one system of betting is used to pay the winnings of the other system, then if one system has a negative expected payout, the other should have a positive expected payout. A positive expected payout cannot be true otherwise casinos would go broke. What retains the casino's advantage is that these two bets are not truly complementary. The Pass Line loses on a roll of 2, 3, or 12. The Don't Pass Line wins on a roll of 2 or 3, and it pushes on a roll of 12. This roll of a 12 is the difference which spoils any advantage that the gambler is looking to find. If you are unfamiliar with the rules of craps, and what was just said made no sense to you, just know that no matter what betting strategy is employed in a fair game, the gambler cannot attain an advantage over the house. The Don't Pass bet will not be referred to any more in this paper. While it does provide great odds for gamblers, it is not a common betting strategy in the casino because of the social implications of the game. A person betting the Don't Pass Line is called a 'wrong better' for a reason. They are hoping to roll a 7 which means that when they win, almost everyone else at the table, the 'right betters,' will lose.

Why Craps?

Among all the games, craps is one which is known to give players one of the best advantages they will see in a casino. More appropriately said, craps gives players one of the smallest disadvantages against the house. After all, casino games are designed so that players will never have the advantage. When rules are followed, dice are fair, and it comes down to straight probabilities, the house will always have an edge. It is advantageous to note that experienced blackjack players can have a small advantage if they employ the optimal strategy while counting cards, but counting cards is, in effect, changing the probabilities of the game due to the fact that the deck is no longer a full deck. Just considering the original probabilities of fair casino games, the house will always have an advantage. Smart gamblers recognize this fact, and they choose to play the games that give the least advantage to the house. Consider the table below:

House Advantages for Popular Casino Games	
Game	***House Advantage***
Roulette (double-zero)	5.3%
Craps (pass/come)	1.4%
Craps (pass/come with double odds)	0.6%
Blackjack - average player	2.0%
Blackjack - 6 decks, basic strategy*	0.5%
Blackjack - single deck, basic strategy*	0.0%
Baccarat (no tie bets)	1.2%
Caribbean Stud*	5.2%
Let It Ride*	3.5%
Three Card Poker*	3.4%
Pai Gow Poker (ante/play)*	2.5%
Slots	5% - 10%
Video Poker*	0.5% - 3%
Keno (average)	27.0%
**optimal strategy*	

(Hannum)

Craps is not as simple as other casino games. Consider Blackjack for example. The player puts down money and bets that he/she can accumulate cards whose sum is higher than the dealer's cards and yet totals to no more than 21. The entire premise of the game can be summed up in a single sentence. Simply by taking a look at a craps table, a beginner player can see that craps is not elementary. There are so many different bets that can happen between "rounds" or even between rolls. If a gambler wants to play with the small house advantage seen in the table above, then he/she has to play the appropriate system. Unfortunately for us (and fortunately for the casinos), the "pass/come" system above is rather complicated. Many craps players are intimidated by the complexity of this game and in turn make elementary bets which simply give the house a greater advantage.

The Beginners' Sucker Bets

Bet	House Advantage
Field	5.56%
Big 6, Big 8	9.09%
2 Proposition, 12 Proposition	13.89%
3 Proposition, 11 Proposition	11.11%
Horn Bet	12.50%
C-E Bet	11.11%
Any Seven	16.67%
Any Craps	11.11%
Hard 4, Hard 10	11.11%
Hard 6, Hard 8	9.09%

See Appendix A for these calculations

Field – Betting that a 2, 3, 4, 9, 10, 11, or 12 will be rolled on the very next roll. Pays 1:1 on 3, 4, 9, and 10. Pays 2:1 on 2 and 12

Big 6 – Betting that a 6 will be rolled before a 7 is rolled. No win/loss for numbers other than 6 and 7. Pays 1:1

Big 8 – Betting that an 8 will be rolled before a 7 is rolled. No win/loss for numbers other than 7 and 8. Pays 1:1

2 Proposition – Betting that 2 will be rolled on the very next roll. Pays 30:1

12 Proposition – Betting that 12 will be rolled on the very next roll. Pays 30:1

3 Proposition – Betting that 3 will be rolled on the very next roll. Pays 15:1

11 Proposition – Betting that 11 will be rolled on the very next roll. Pays 15:1

Horn Bet – Four bets in one. A $4 Horn Bet is the same as putting $1 on each of the 2, 3, 11, 12 propositions. Payout is what is listed for each individual proposition

C-E Bet – "Craps, Eleven Bet" – Similar to the Horn Bet in that gamblers want to roll a 2, 3, 11, or 12 however pays 3:1 on the 2, 3, 12, and pays 7:1 on the 11

Any Seven – Betting that a 7 will be rolled on the very next roll. Pays 4:1

Any Craps – Betting that a 2, 3, or 12 will be rolled on the very next roll. Pays 7:1

Hard 4 – "Hard" refers to rolling the same number on each dice. This is a bet that a [2, 2] will be rolled before a [1, 3], [3, 1], or a 7 is rolled. Wins on [2, 2] and loses on "other" 4's and any 7. Pays 7:1

Hard 10 – Betting that a [5, 5] will be rolled before all "other" 10's and any 7. Pays 7:1

Hard 6 – Betting that a [3, 3] will be rolled before all "other" 6's and any 7. Pays 9:1

Hard 8 – Betting that a [4, 4] will be rolled before all "other" 8's and any 7. Pays 9:1

Example: The Field Bet

When a player bets the Field, he/she is betting the sum of the next roll will be one of the numbers listed in the Field, namely 2, 3, 4, 9, 10, 11, or 12. The payout is 1-1 for all numbers except 2 and 12. As denoted by the circle seen on the craps layout, these two numbers payout at 2-1. Notice however what numbers are missing from the Field bet. A player can roll anything from two through twelve, and almost all of the middle numbers are excluded. Whether calculating the numbers or just relying on personal experience with dice, one can see that the middle numbers are the most common rolls! The house clearly has an advantage on this bet, but how bad is it?

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(payout)
2	1	2.78%	2	0.056
3	2	5.56%	1	0.056
4	3	8.33%	1	0.083
5	4	11.11%	-1	-0.111
6	5	13.89%	-1	-0.139
7	6	16.67%	-1	-0.167
8	5	13.89%	-1	-0.139
9	4	11.11%	1	0.111
10	3	8.33%	1	0.083

11	2	5.56%	1	0.056
12	1	2.78%	2	0.056
Total		**100%**		**-0.0556**

(a) Number of ways to roll the corresponding number
(b) (a) / 36
(c) Payout given the roll
(d) (b) * (c)

On average, the Field bet results in a loss of $0.0556 for every $1 bet. This means that the house advantage for the field bet is 5.556%. This is a significant advantage that the player is giving to the casino. Unfortunately, the Field bet is made far too often because it is a simple bet with immediate results. It is easily understood by new craps players and the win or loss will occur after just one roll of the dice.

Craps Terminology

Shooter	The person who rolls the dice
Come Out Roll	The first roll of the dice in a betting round
Establishing a Point	Rolling a 4, 5, 6, 8, 9, 10 on the Come Out Roll.
Point Number	The point that is established on the Come Out Roll (4, 5, 6, 8, 9, or 10)
Making the Point	Rolling the Point Number before Rolling a 7
Seven Out	Rolling a 7 before Making the Point
Come Bet	A "Virtual Pass Line Bet" but is made after the Come Out Roll
Come Point	The number serving as a point for a Come Bet

The Pass Line Bet

Craps can be one of the most complicated and fast-paced games in a casino. Players can make multiple bets on a single roll of the dice. They can change their bets between rolls. Payouts differ based on not only the bet but also the number shown on the dice. Here, the various side-bets that a play can make will be excluded. We shall start with the basics. At a high level, the objective in craps is to establish a point (4, 5, 6, 8, 9, or 10) and to roll that number a second time prior to rolling a 7. For example, it is Steve's turn to roll the dice. He places $10 on the Pass Line. In effect, Steve is betting that he will be able to roll the same number that he rolls on his first roll a second time prior to rolling a 7. He is betting that he will roll the established Point Number before Sevening Out. Consider the following sequences of Steve's rolls:

6	Point is Established	
6	Made the Point	Win $10

8	Point is Established	
2		
5		
9		
4		
2		
2		
11		
8	Made the Point	Win $10

5	Point is Established	
3		
8		
7	Seven Out	Lose $10

Before considering all of the various combinations of side-bets, there are certain nuances that affect the Come Out Roll. The Come Out Roll is the first roll of the dice in a betting round. A point is established when a 4, 5, 6, 8, 9, 10 is rolled. What about the other numbers? If the shooter throws a 2, 3, or 12 on the Come Out Roll, this is "Craps," and the player loses what was bet on the Pass Line. If the shooter throws a 7 or 11 on the Come Out Roll, it is called a "Natural." The shooter wins what is on the Pass Line. This is the only time that a player wants to see a 7 be rolled. For the rest of the game, players resent that number so much so that it is considered bad luck to even say "seven" at the table. Consider the following sequence of Steve's rolls starting with the Come Out Roll:

7	Natural	Win $10
2	Craps	Lose $10
10	Point is Established	
3		
11		
7	Seven Out	Lose $10
12	Craps	Lose $10

The Odds Bet

The Odds Bet is what makes the game of craps so unique. Gamblers will not find a better bet in the rest of the casino! Taking advantage of this bet is how gamblers can lower the house advantage even more. The Odds Bet is given the name that it has because its payout is at even odds. This means that if a person is twice as likely to lose (66.67% chance of losing and 33.33% chance of winning), then a win will pay $2 for every $1 bet. The Odds bet is slightly more complicated because the payout is not simply 2:1 for each Point Number. Its payout changes depending on the probability of repeating the number that has already been established as the Point Number. Take a look at the following table of payouts.

Point Number	Probability of Repeating before Rolling a 7	Payout Ratio	Payout on $1
4	3/6	2:1	$2.00
5	4/6	3:2	$1.50
6	5/6	6:5	$1.20
8	5/6	6:5	$1.20
9	4/6	3:2	$1.50
10	3/6	2:1	$2.00

This can get complicated, so consider the following examples. Once again, they assume an original bet of $10 is made on the Pass Line.

5	Point is Established	Bet an additional $10 on Odds
5	Made the Point	Win $10 from Pass Line + $15 for Odds

4	Point is Established	Bet an additional $10 on Odds
6		
5		
10		
4	Made the Point	Win $10 from Pass Line + $20 for Odds

8	Point is Established	Bet an additional $10 on Odds
8	Made the Point	Win $10 from Pass Line + $12 for Odds

8	Point is Established	Bet an additional $10 on Odds
6		
5		
7	Seven Out	Lose $10 from Pass Line and $10 from Odds

As mentioned above, the Odds Bet eliminates the edge that the casino has on gamblers, and this is because of the even odds payout. When the payout is at even odds, a gambler's expected profit/loss for this bet is $0. The house has no advantage whatsoever! This is the best bet in the game of craps, so why must we bother with the Pass Line?

The casinos have to take back the advantage somehow. They do so because we cannot simply make an Odds Bet whenever we feel like it. There are two things that protect the casino advantage. The first is that Odds can only be taken on bets that are already on the Pass Line. In effect, the Odds Bet cannot be the only bet that a person has on the table. By requiring that there is at least some bet already sitting on the Pass Line, gamblers are required to have more "skin in the game." Additionally, this extra bet on the Pass Line does not pay out at even odds. This retains the house advantage on this bet. The second rule that protects the casino advantage is that they limit the amount that a gambler can bet as Odds. Most casinos permit Double Odds. This means that if x is bet on the Pass Line, then a maximum of 2x can be bet as Odds. While the rule of thumb is that casinos allow Double Odds, each casino has the authority to make its own rules. Notice how the casino advantage decreases as the permissible Odds increases.

House Advantage on Pass Line Bets

Odds Taken	House Advantage
0x	1.410%
1x	0.848%
2x	0.606%
3x	0.471%
5x	0.326%
10x	0.184%

20x	0.099%
100x	0.021%

(Bluejay)

The limit as the Odds increase to infinity of the house advantage is 0. Said another way, as the Odds increase, the house advantage decreases. The house advantage will never equal 0 exactly, and unfortunately, no amount of Odds can ever give the craps player an advantage over the casino. Consider an example out of the classroom. A student takes the first test of the semester and receives a 90%. If the student receives a 100% on each and every test throughout the rest of the semester, how will this student's average change?

(a) Number of Tests	(b) Sum of All Grades	(c) Average
2	190	95.00%
3	290	96.67%
5	490	98.00%
10	990	99.00%
50	4,990	99.80%
100	9,990	99.90%
1000	99,990	99.99%

(a) Chosen arbitrarily
(b) 90 + 100*[(a) - 1]
(c) (b) / [100*(a)]

According to the table, if there are 50 tests throughout the entire semester, the student's grade will round up to a 100%. However it will never average to 100% exactly, and of course, the average will never come out to anything over 100%. The same type of understanding applies to the Odds Bet in craps. A player can enhance his chances by taking more Odds, but he cannot level the playing field exactly.

The Come Bet

The Come Bet is an additional bet, or a side-bet, that can be made while in the midst of a game that is already in progress. The Come Bet works exactly like the Pass Line Bet with one exception. Players bet on the Pass Line for the Come Out Roll only; they can make a Come Bet at any point during a series of rolls. Other than that, the Come Bet functions just like the Pass Line Bet. It loses on a roll of 2, 3, or 12. It wins on a roll of 7 or 11. If none of these numbers are rolled, the Come Bet will win when the value shown on the dice is rolled a second time prior to rolling a 7.

		Bet $10 on Pass Line
10	Point is Established	Make $10 Come Bet
5	5 is now a Come Point	
3		
5	Made the Come Point Win	$10 on the Come Bet
6		
10	Made the Point	Win $10 on the Pass Line

		Bet $10 on Pass Line
3	Craps	Lose $10, Put another $10 bet on Pass Line
12	Craps	Lose $10, Put another $10 bet on Pass Line
6	Point is Established	Make $10 Come Bet
7	Seven Out	Lose $10 on Pass Line, Win $10 on Come Bet

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
ROLL	**# Ways**	**Prob**	**Payout**		**Outcome**	**Prob**	**Payout**	**Exp(Profit)**
2	1	1/36	-1		Craps			-0.0278
3	2	2/36	-1		Craps			-0.0556
4	3	3/36		6 ways to roll 7 and lose	7 Out	6/9	-1	-0.0556
				3 ways to roll 4 and win	Made the Point	3/9	1	0.0278
5	4	4/36		6 ways to roll 7 and lose	7 Out	6/10	-1	-0.0667
				4 ways to roll a 5 and win	Made the Point	4/10	1	0.0444
6	5	5/36		6 ways to roll 7 and lose	7 Out	6/11	-1	-0.0758
				5 ways to rolls a 6 and win	Made the Point	5/11	1	0.0631
7	6	6/36	1		Natural			0.1667
8	5	5/36		6 ways to roll 7 and lose	7 Out	6/11	-1	-0.0758
				5 ways to rolls a 6 and win	Made the Point	5/11	1	0.0631
9	4	4/36		6 ways to roll 7 and lose	7 Out	6/10	-1	-0.0667
				4 ways to roll a 5 and win	Made the Point	4/10	1	0.0444
10	3	3/36		6 ways to roll 7 and lose	7 Out	6/9	-1	-0.0556
				3 ways to roll 4 and win	Made the Point	3/9	1	0.0278
11	2	2/36	1		Natural			0.0556
12	1	1/36	-1		Craps			-0.0278
	36	**1.00**						**-0.0141**

(a) Number of ways to roll the corresponding number
(b) (a) / 36
(c) Payout given that you roll a Craps or a Natural
(f) (d) / Sum of the two rows in (d)
(g) Payout on a win or a loss after a point is established
(h) (b) * (c)
(h) (b) * (f) * (g)

The house advantage for a bet on the Pass Line is 1.41%. This is the value that was given above which compared the game of craps to other casino games. 1.41% is among the best odds that a gambler will face inside of a casino, but remember the concept of Double Odds? Double Odds reduces the house advantage even further!

Most casinos offer Double Odds, and a system playing the Pass Line and always taking odds reduces the house advantage to 0.61%. The interesting thing to note is that for a given round of craps, the expected loss is 1.41% on the Pass Line bet. However, the value of house advantage is a calculation based on a total bet. Allow me to dispel this even further. According to probability, a craps player can expect to lose 1.41% of what is bet on the Pass Line, but he also has money that is bet as Odds. We already know that for the Odds bet alone, the expected loss is 0.0 because Odds pay out at even odds. Therefore, for the given round of craps with X bet on the Pass Line and 2X bet as Odds, he can expect to lose 1.41% + 0% + 0% = 1.41%. Now it is time to calculate house advantage, so the amount bet must be taken in to consideration. To start the game, a bet must be placed on the Pass Line. There is 100% certainty that this bet is made. $10 is on the Pass Line. The dice are rolled for the first time. This is the Come Out Roll. Then, if a point is established (if the roll is 4, 5, 6, 8, 9, or 10), then the craps player will take Double Odds. In this case, $20 is placed as Odds, but this only happens when a point is established. A point is established only 66.6% of the time. In essence, your total bet will be $10 one third of the time, and your total bet will be $30 two thirds of the time. In this example, your expected bet is 10/3 + 30*(2/3) = $23.3. The table below illustrates this idea for the $1 example.

Double Odds

	Bet	P(that Bet)	Exp Bet
Natural or Craps	1	33%	0.333333333
Point is Established	3	67%	2
Total		**100%**	**2.333333333**

1.41% / 2.33 = -0.61%

Optimal Strategy Simulation

The screenshot below is taken from an Excel spreadsheet. The three columns on the left are the simulated rounds of craps. The boxes on the right side of the picture are the inputs for the simulation and an analysis of the results. When the user plugs in his/her desired inputs and clicks on the picture, the simulation begins. The coding behind the scenes is run in Visual Basic for Applications (VBA). The VBA code relies on random number generators to imitate the rolls of the dice and, in turn, mimic a real game of craps.

Win/Loss	# Rolls	Win/Loss per $1 Bet
96	13	0.457142857
-30	12	-0.375
34	11	0.34
-40	7	-0.8
-33	6	-0.507692308
-10	2	-0.5
-50	9	-1
-35	5	-0.777777778
-33	10	-0.275
-30	4	-0.75
-10	2	-0.5
0	4	0
-10	2	-0.5
-15	3	-0.6
2	7	0.028571429
-10	2	-0.5
-25	10	-0.3125
10	19	0.054054054
-5	3	-0.2
15	12	0.15
-3	10	-0.025

Bet	5
Max #s working	3
Raise Amt	5
Raise Every __ Hits	2
# Rounds to Play	75000

Inputs

Net Gain/Loss	-42874
Avg Gain/Loss	-0.57165
Avg # Rolls	8.523027
% Wins	26%
Max Gain	1905
Max Loss	-81
Max # Rolls	91
Gain/Loss per $1	-0.00627
Variance	3264.745

Analysis of Results

Premise of the Simulation

The craps simulator only utilizes one betting strategy. It automatically makes a Pass Line bet to get the round of craps started. Then, according to user inputs, it makes Come Bets at the appropriate times. It automatically takes Double Odds on each of these bets. This is the strategy discussed above which reduces the house advantage to 0.61%.

In general, craps is a game of a lot of little losses, which hopefully can be compensated with a few, big gains. The real question is if the craps player is patient enough to realize this and not get discouraged along the way. They must also bring enough capital to the craps table to withstand these losses so that they are still playing when the dice get hot. When the dice get hot, this is when the player has to take advantage. Some strategies say that you need to increase your bets during these hot streaks. Ultimately, we know that each roll of the dice is an independent event, and the probabilities never change whether the dice are hot or they are cold. However we are human beings, and we like to believe in luck and superstition. In this Excel simulation of craps, I've given the user the ability to increase the bet during these hot streaks.

The Inputs

Bet – This is the amount that you bet on the Pass Line. This initial bet also dictates how much is bet in Odds since Odds are set at two times the Pass Line bet.

Max #s Working – If I roll a 6 on the Come Out Roll, then I of course want to roll another 6, and I will win. If I make a Come Bet and then proceed to roll an 8 on the second roll, then I will be hoping to roll a 6 or an 8, and I will be a winner. I call this having two numbers working for me since I have the possibility of winning on the Point

Number or the Come Point. If I made a second Come Bet and proceeded to roll a 9, then I would have three numbers working for me (6, 8, and 9).

Raise Amt – If I think I am in the middle of a hot streak as described above, then I might think about increasing my bet (and hopefully increasing my winnings). "Raise Amt" is the additional amount that I will bet on top of the normal bet. In the screenshot above, Bet = 5 and Raise Amt = 5. This means that the first time I increase my bets, I would be wagering $5 + $5 = $10. Consequently, I would be taking 2*$10 = $20 in Odds bets.

Raise Every __ Hits – This is the part where you can determine what qualifies as a hot streak. If in the middle of a round, I repeat a number (Make the Point), I call this a hit. If I repeat two numbers, I have two hits. I will now be betting $5 + $5 = $10. If my hot streak continues and my two hits turns into four hits, then I will be bettering $10 + $5 = $15. This streak continues until I roll a 7. When a roll a 7, my "hit count" starts over at zero, and my bet would return to the initial betting amount ($5 for this example).

 # Rounds to Play – A round starts with the first Come Out Roll, and a round ends with the first Seven Out. As soon as I Seven Out, the gain/loss is totaled up, and the new round begins with the first Come Out Roll. The Excel macro takes about .6 to .7 seconds to run every 1000 simulations.

The Results

For this specific simulation that I ran in the screen shot above, I had the macro run 75,000 rounds of craps. After playing this many rounds, the simulator says that I would have lost $42,875. This may seem like a bad night at the craps table, but is it really? By summing the second column of data, these 75,000 rounds equated to 639,227 rolls of the dice. At a live craps table, the speed of the game depends on a number of factors which include number of players at the table and adeptness of the casino staff working the table. The number of rolls per hour can vary from 70-100 rolls per hour. If we give the casino the benefit of the doubt and assume 100 rolls per hour, these 75,000 rounds would take 639,227/100 = 6,392 hours. This is more than 266 days of straight craps! Again, $42,875 may seem like a lot of money, but when spread out across this much time, it is rather insignificant.

Craps is a game of a lot of small losses which are (hopefully) offset with a few, large wins. In the above simulation, we only came out ahead on 26% of our rounds of craps. Notice, that on average, each round resulted in a $0.57 loss. Also notice the maximum gain from these 75,000 rounds. One round lasted 91 rolls, and we profited more than $1,900 from that single round! This game alone would have taken nearly an hour to play in real life! It is these big rounds that make craps worth playing. This is what keeps players coming back for more!

The next item to notice is the "Gain/Loss per $1." In essence, this is the house advantage for the simulated rounds of craps. This calculation is the net gain/loss for all rounds, divided by the total amount bet for all rounds. The net gain/loss is simple; it is the sum of the first column of numbers. To calculate the total amount bet, I have a variable inside the code of the macro that adds each bet as it is placed on the table. Since we are playing craps and taking double odds, in a perfect world, we expect the house to have a 0.61% advantage. For the given 75,000 rounds above, the house advantage was calculated to be 0.63%. This is a phenomenal result! If we ran it again, this value may or may not be as close to the true house advantage. However, if we were to increase "#

Rounds" to 100,000 or 150,000, we might expect to see our calculated house edge to be even closer to the true value of 0.61%.

The calculated variance does not make a lot of logical sense for a single simulation, but it is interesting to watch how the variance changes when you change the user inputs. For example, consider the "Raise Amt" which dictates if we increase our bets during hot streaks. If we decide to keep our bet constant, we set 'Raise Amt" equal to 0. Our variance will be significantly smaller than if "Raise Amt" equals 5. Obviously, if we increase our bets, we will increase our wins and increase our losses, and this in turn results in a larger variance.

Using Multiple Linear Regression to Predict Percentage of Wins

As mentioned above, the craps game simulator is designed to only make Pass Line bets, make Come bets, and take Odds on these bets. As we know from the mathematical analysis, this betting strategy has a house advantage of 0.61%. Even though there are different "inputs" that we can plug into the simulator, these "inputs" only affect the amount that we are betting, the number of bets to take, and the option to increase our bets during the course of the game. No matter how we change these "inputs," we should still expect to see a house advantage near 0.61%.

As an example of this, consider buying a scratch off lottery ticket. For this example, let the ticket costs $1; let it have only one payout, which is $5; let the probability of winning be 25%. The house advantage is the expected payout minus the cost of the scratch off, divided by the amount that you spent. This puts the answer in terms of house advantage *per amount gambled.*

$$House\ Advantage = \frac{(5 * .25) - 1}{1} = \frac{1.25 - 1}{1} = 0.25$$

Consider buying ten scratch offs this time. The calculation of house advantage is as follows

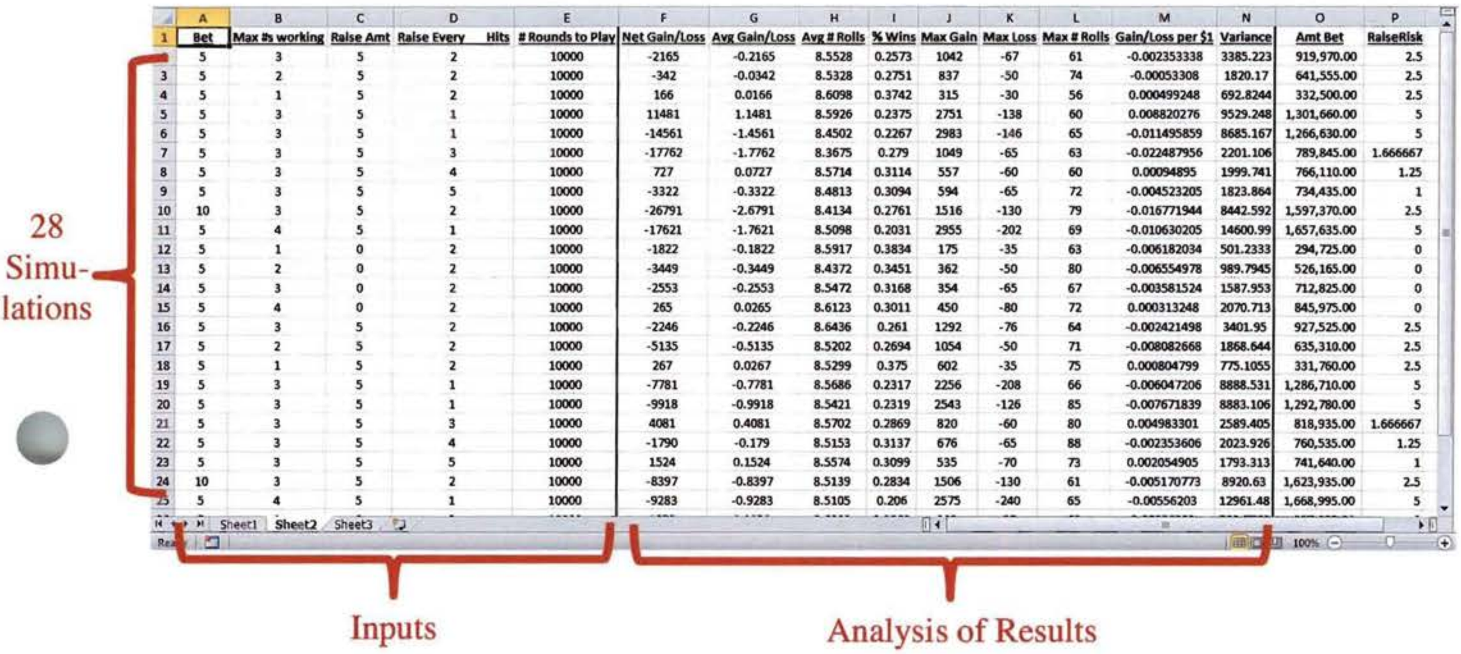
$$House\ Advantage = \frac{10 * (5 * .25) - 10}{10} = \frac{12.5 - 10}{10} = \frac{2.5}{10} = 0.25$$

The house advantage remains the same. What changes is the potential win and the potential loss. When buying just a single lottery ticket, the most I can lose is $1, and the most I can win is $5. Upon buying ten tickets, I could lose as much as $10; I could make $50; or there could a lot of other outcomes somewhere in between.

The more scratch off tickets that are bought, the greater the variability of the outcome. The greater the variability, the greater the risk you are taking on. Similarly with craps, the betting strategy can lead to greater or lesser amounts of risk. One way to analyze risk is to consider the percentage of wins. One round of craps is considered the time that a shooter holds the dice or rather the time until the shooter Sevens Out. Some rounds will yield a profit. Some rounds will yield a loss. Percentage of wins is defined as

$$\%\ Wins = \frac{\#\ rounds\ yielding\ a\ profit}{total\ \#\ rounds\ simulated}$$

In order to accumulate a dataset, I ran 28 simulations. Each simulation randomly computed 10,000 trials (rounds) of craps. Of the 28 simulations, there are 13 different types of betting strategies. That is, there are 13 different combinations of inputs. The goal is to find a regression model that uses the input variables to predict percentage of wins.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Bet	Max #s working	Raise Amt	Raise Every Hits	# Rounds to Play	Net Gain/Loss	Avg Gain/Loss	Avg # Rolls	% Wins	Max Gain	Max Loss	Max # Rolls	Gain/Loss per $1	Variance	Amt Bet	RaiseRisk
	5	3	5	2	10000	-2165	-0.2165	8.5528	0.2573	1042	-67	61	-0.002353338	3385.223	919,970.00	2.5
3	5	2	5	2	10000	-342	-0.0342	8.5328	0.2751	837	-50	74	-0.00053308	1820.17	641,555.00	2.5
4	5	1	5	2	10000	166	0.0166	8.6098	0.3742	315	-30	56	0.000499248	692.8244	332,500.00	2.5
5	5	3	5	1	10000	11481	1.1481	8.5926	0.2375	2751	-138	60	0.008820276	9529.248	1,301,660.00	5
6	5	3	5	1	10000	-14561	-1.4561	8.4502	0.2267	2983	-146	65	-0.011495859	8685.167	1,266,630.00	5
7	5	3	5	3	10000	-17762	-1.7762	8.3675	0.279	1049	-65	63	-0.022487956	2201.106	789,845.00	1.666667
8	5	3	5	4	10000	727	0.0727	8.5714	0.3114	557	-60	60	0.00094895	1999.741	766,110.00	1.25
9	5	3	5	5	10000	-3322	-0.3322	8.4813	0.3094	594	-65	72	-0.004523205	1823.864	734,435.00	1
10	10	3	5	2	10000	-26791	-2.6791	8.4134	0.2761	1516	-130	79	-0.016771944	8442.592	1,597,370.00	2.5
11	5	4	5	1	10000	-17621	-1.7621	8.5098	0.2031	2955	-202	69	-0.010630205	14600.99	1,657,635.00	5
12	5	1	0	2	10000	-1822	-0.1822	8.5917	0.3834	175	-35	63	-0.006182034	501.2333	294,725.00	0
13	5	2	0	2	10000	-3449	-0.3449	8.4372	0.3451	362	-50	80	-0.006554978	989.7945	526,165.00	0
14	5	3	0	2	10000	-2553	-0.2553	8.5472	0.3168	354	-65	67	-0.003581524	1587.953	712,825.00	0
15	5	4	0	2	10000	265	0.0265	8.6123	0.3011	450	-80	72	0.000313248	2070.713	845,975.00	0
16	5	3	5	2	10000	-2246	-0.2246	8.6436	0.261	1292	-76	64	-0.002421498	3401.95	927,525.00	2.5
17	5	2	5	2	10000	-5135	-0.5135	8.5202	0.2694	1054	-50	71	-0.008082668	1868.644	635,310.00	2.5
18	5	1	5	2	10000	267	0.0267	8.5299	0.375	602	-35	75	0.000804799	775.1055	331,760.00	2.5
19	5	3	5	1	10000	-7781	-0.7781	8.5686	0.2317	2256	-208	66	-0.006047206	8888.531	1,286,710.00	5
20	5	3	5	1	10000	-9918	-0.9918	8.5421	0.2319	2543	-126	85	-0.007671839	8883.106	1,292,780.00	5
21	5	3	5	3	10000	4081	0.4081	8.5702	0.2869	820	-60	80	0.004983301	2589.405	818,935.00	1.666667
22	5	3	5	4	10000	-1790	-0.179	8.5153	0.3137	676	-65	88	-0.002353606	2023.926	760,535.00	1.25
23	5	3	5	5	10000	1524	0.1524	8.5574	0.3099	535	-70	73	0.002054905	1793.313	741,640.00	1
24	10	3	5	2	10000	-8397	-0.8397	8.5139	0.2834	1506	-130	61	-0.005170773	8920.63	1,623,935.00	2.5
25	5	4	5	1	10000	-9283	-0.9283	8.5105	0.206	2575	-240	65	-0.00556203	12961.48	1,668,995.00	5

Dependent Variable:	% Wins
Independent Variables:	Bet Max #s Working Raise Amt Raise Every __ Hits

One problem that can occur here is that if "Raise Amt" is zero, then it does not matter what value we assign to "Raise Every __ Hits." After all, it does not matter how often we increase our bet if each time we increase it, we increase it by $0. There is some ambiguity between these two variables. To correct for this, I created my own variable called "RaiseRisk." "RaiseRisk" is an interaction variable between "Raise Amt" and

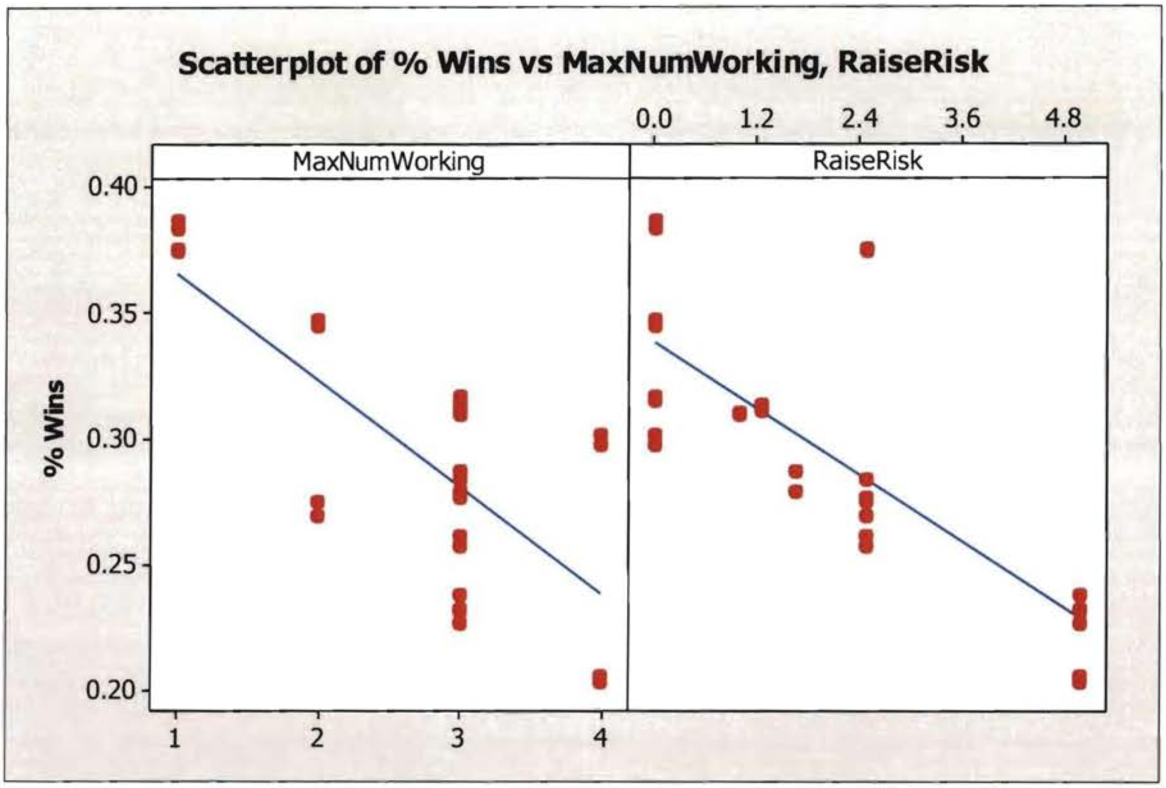
"Raise Every __ Hits." It is a variable which combines the amount of each raise and the frequency of raising. The equation used is as follows:

$$RaiseRisk = \frac{Raise\ Amt}{Raise\ Every\ \_\ Hits}$$

This equation eliminates the potential errors cause by the previous two variables. As "Raise Risk" increases, the more you will be increasing your bets. Conversely, the lower the value for "Raise Risk," the less frequently you are increasing your bets, and the more conservatively you are playing. Now, the regression model will be made up of:

Dependent Variable:	% Wins
Independent Variables:	Bet Max #s Working RaiseRisk

After running some tests in MINITAB, we can conclude that "Bet" is not significant in the model, and it can be discarded. We will end up using only "Max #s Working" and "RaiseRisk." Before we consider the final model, we can see that both variables independently exhibit a downward trend. That is, as the values for the independent variables increase, the values for "% Wins" tend to decrease.



Next, we can combine the two independent variables into a single Multiple Linear Regression Model to predict percentage wins. We get the following MINITAB output:

Regression Analysis: % Wins versus MaxNumWorking, RaiseRisk

```
The regression equation is
% Wins = 0.420 - 0.0330 MaxNumWorking - 0.0178 RaiseRisk


Predictor           Coef    SE Coef       T      P
Constant         0.41965    0.01010   41.57  0.000
MaxNumWorking  -0.033037   0.003640   -9.07  0.000
RaiseRisk      -0.017780   0.001776  -10.01  0.000


S = 0.0163856   R-Sq = 90.8%   R-Sq(adj) = 90.0%


Analysis of Variance

Source          DF        SS        MS       F      P
Regression       2  0.066078  0.033039  123.06  0.000
Residual Error  25  0.006712  0.000268
Total           27  0.072790
```

The p values of 0.000 indicate that "Max #s Working" and "RaiseRisk" are significant in predicting "% Wins," and the one at the bottom, in the Analysis of Variance section, indicates that our overall model is effective in predicting "% Wins." The R-Sq value of 90.8% tells us that this model accounts for about 90.8% of the variability in the "% Wins" variable. Both of these we like to see. The MINITAB output indicates that we can be confident that this model is an effective predictor of percentage of wins. The final regression equation is:

```
% Wins = 0.420 - 0.0330 MaxNumWorking - 0.0178 RaiseRisk
```

If "Max #s Working" and "RaiseRisk" equal zero (if we are taking absolutely no risk), this regression equations says that we will win approximately 42% of the time. Notice that when we increase the risk we are taking and increase the values for "Max #s Working" and "RaiseRisk," the percentage of wins only decreases. If we play conservatively and only put money on the Pass Line, play no Come Bets, and never increase our bets, we might expect to win about 38.7% of the time (.42 - .033 = .387). If we play the Pass Line, make two Come Bets, and never increase our bet, the regression equation indicates that we might win about 32% of the time (.42 - .033*3 = .321). If we raise our bets during the game, we win even less frequently!

The table below outlines prediction values and prediction intervals for "% Wins" based on the above regression model. The left hand side represents betting strategy. Row 1 just a Pass Line bet and never increasing the bet. Row 2 is a strategy consisting of a maximum of one Pass Line bet and two Come bets at the same time and also never increasing the amount bet. Row 3 is one Pass Line bet and up to two Come bets while increasing the amount bet by (on average) $1.66 for every point that you make during the course of one continued round of craps. The "Fit" column is the point prediction. For the given betting strategy in Row 1, the regression model indicates that the average of the percentage of games won will be about 38.661%. "SE Fit" is the standard error of this measurement. "95% CI" is the 95% confidence interval for this predicted value. This means that the interval from 37.203% to 40.120% will contain the true value of the *average* win percentage 95% of the time. Similarly, "95% PI" is the 95% prediction interval for this predicted value. This means that the interval from 34.985% to 42.338% will contain the true value of *a single simulation's* win percentage 95% of the time. Again, the correlation between riskiness and winning percentage is apparent. The betting strategy for Row 3 is obviously more risky than that of Row 1 because it has more wagers on the table at any given time. As expected, the decimal values for percentage of wins in Row 3 are lower than those of Row 1 and Row 2.

Betting Strategy		**Prediction Point and Intervals for "% Wins"**			
Max #s Working	RaiseRisk	Fit	SE Fit	95% CI	95% PI
1	0	0.38661	0.00708	(0.37203, 0.40120)	(0.34985, 0.42338)
3	0	0.32054	0.00511	(0.31002, 0.33106)	(0.28519, 0.35589)
3	1.66	0.29085	0.0034	(0.28385, 0.29784)	(0.25638, 0.32531)

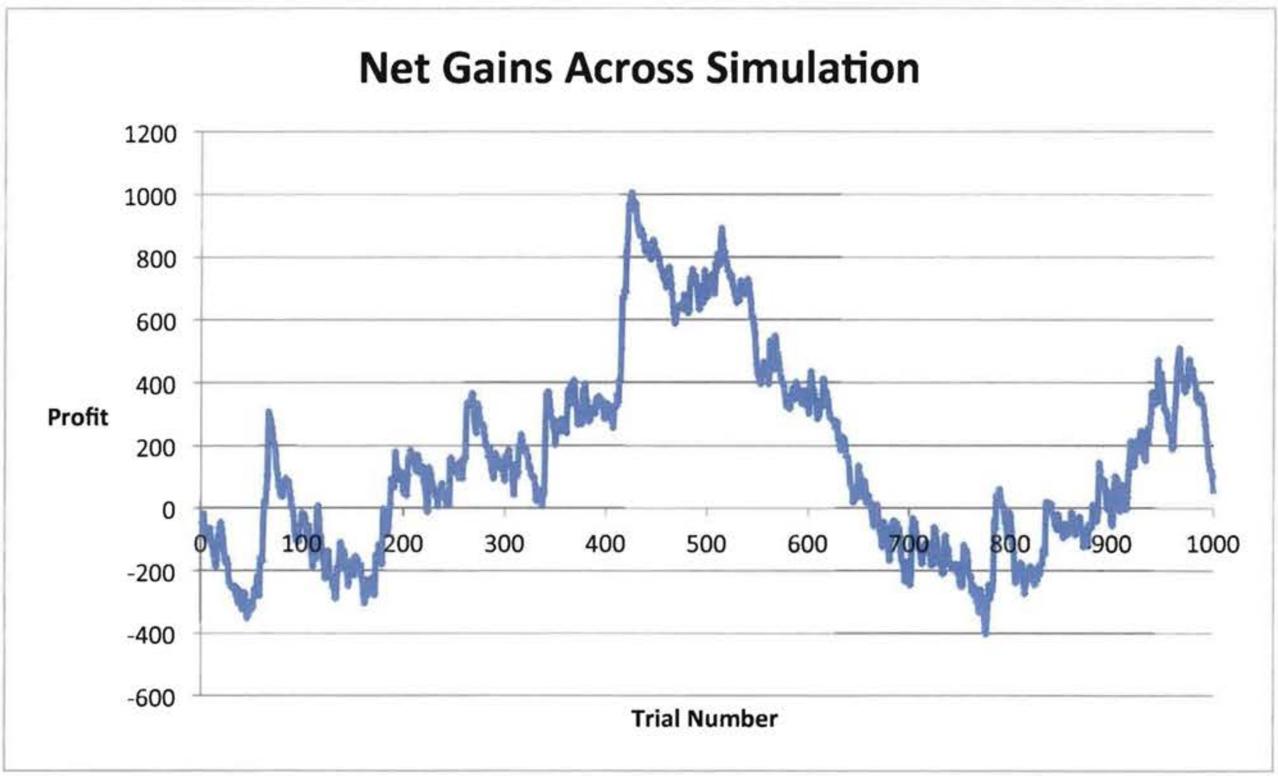
The question exists, why would anyone take on more risk and also win less frequently? Remember that the house advantage remains the same throughout this entire process. The answer is that a risk taker will win less frequently, but his/her wins will be of more significant amounts! Playing conservatively, a craps player may win 40% of the rounds of craps, but the average win may only be $5. If a risk taker only wins 25% of the time, but his/her average win is $25, then this gamble may be worth the risk!

When a craps player uses the optimal strategy, the house advantage remains at 0.61%. This will not change. What changes is the variability. A conservative gambler may want to limit the potential loss and only buy one scratch off. Others have the "go big or go home" attitude and may choose to buy fifty scratch offs. The gambler has to evaluate his/her own motivation. Are you just playing for fun and therefore you should just play conservatively? Are you playing to make a huge profit? If so, are you willing to accept the possibility of a large loss? No matter how risky one desires to play this game, the probabilities remain the same. The house always has the advantage. Some days you will get lucky and beat the house; some days you will not.

Tracing Net Profit throughout a Session at the Craps Table

Gambling is about knowing when to walk away. Greed has a funny way of making it difficult to walk away from the table. The natural fluctuation of wins and losses that go with gambling mean that it is as much about timing as it is luck. It has already been established that, in the long run, the house will always win. The key is being able to walk away during the times when luck allows a person to overcome the house advantage.

The craps game simulator was used to generate 1000 trials. The profit or loss for each trial was summed together in order to create a running total. This running total of net gain was put in graphical form. The graph below shows the natural fluctuation between wins and losses. In the simulation below, the final result after 1000 rounds of craps was a profit of $56. However, during the course of those simulated trials, the craps player was up as much as $1006 and down as much as $403. The final number of $56 just so happened to correspond with the 1000th trial and so became to final net gain.



Throughout the 1000 simulated trails depicted in the graph, there were a total of 8760 rolls of dice. As was done before, we can roughly convert this value into real time.

At a live craps table, the number of rolls per hour can range from 70-100 rolls per hour. Again, we will give the casino the benefit of the doubt and assume that they function at 100 rolls per hour. The above simulation would take 87.6 hours which is more than three and a half days of continuously playing craps. In reality, this 3.5 day span of time could represent an afternoon's session at the craps table, or it could represent a lifetime of sessions at the craps table. Everyone will experience ups and downs over the course of the hours, days, and even years spent at the craps table. Unlike the graph above, it is not so simple to see what is a small streak of bad luck and what is a complete and utter, downward spiral. No one can ever know when would actually be the ideal time to walk away from the table. In the end, this form of entertainment is so challenging because rolling the dice is a gamble, and timing is a gamble.

Conclusion

Ultimately, there is no secret betting strategy for craps that will enable gamblers to make a profit in the long run. The purpose of this paper is more to help gamblers limit their future losses. If you have a weekend trip to Las Vegas planned and you plan to spend some of that time gambling, consider the house advantages of what you will be playing. Consider playing craps. Avoid the sucker bets; do not give the house any more advantage than necessary. Playing the Double Odds Pass/Come strategy reduces the house advantage to 0.61%. In the long run, mathematics proves that the casino will always win, but for an afternoon of gambling you could get lucky! Remember too that gambling is a form of entertainment. Just as we would pay to go to a movie or pay to attend a sporting event, we should expect to pay to go to the casino. If you are going to spend the afternoon gambling, remember the mathematics. Choose a game with the smallest house advantage. Avoid the sucker bets. Stick to the system that puts your money to best use. Above all, play responsibly, and have fun.

Appendix A

Field Bet

Roll	(a) # Ways	(b) P(Roll)	(c) Payout	(d) E(profit)
2	1	2.78%	2	0.056
3	2	5.56%	1	0.056
4	3	8.33%	1	0.083
5	4	11.11%	-1	-0.111
6	5	13.89%	-1	-0.139
7	6	16.67%	-1	-0.167
8	5	13.89%	-1	-0.139
9	4	11.11%	1	0.111
10	3	8.33%	1	0.083
11	2	5.56%	1	0.056
12	1	2.78%	2	0.056
Total		**100%**		**-0.0556**

(a) Number of ways to roll the corresponding number
(b) (a) / 36
(c) Payout given the roll
(d) (b) * (c)

Big 6

**Same procedure for Big 8

Roll	(a) # Ways	(b) P(Roll)	(c) Payout	(d) E(profit)
7 before 6	6	54.55%	-1	-0.545
6 before 7	5	45.45%	1	0.455
Total		**100%**		**-0.0909**

(a) Number of ways to roll the corresponding number
(b) (a) / (6+5)
(c) Payout given the roll
(d) (b) * (c)

2 Proposition

**Same procedure for 12 Proposition

(a) (b) (c) (d)

Roll	# Ways	P(Roll)	Payout	E(profit)
2	1	2.78%	30	0.833
Other	35	97.22%	-1	-0.972
Total		**100%**		**-0.1389**

(a) Number of ways to roll the corresponding number
(b) (a) / (36)
(c) Payout given the roll
(d) (b) * (c)

3 Proposition

**Same procedure for 11 Proposition

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(profit)
3	2	5.56%	15	0.833
Other	34	94.44%	-1	-0.944
Total		**100%**		**-0.1111**

(a) Number of ways to roll the corresponding number
(b) (a) / (36)
(c) Payout given the roll
(d) (b) * (c)

Horn Bet

	(a)	(b)	(c)	(d)	(e)	(f)
Roll	# Ways	P(Roll)	Bet	Payout	Profit/Loss	E(profit)
2	1	2.78%	1	30	27	0.750
3	2	5.56%	1	15	12	0.667
11	2	5.56%	1	15	12	0.667
12	1	2.78%	1	30	27	0.750
Other	30	83.33%	0	0	-4	-3.333
Total		**100%**	**4**			**-0.5000**
						-0.125

(a) Number of ways to roll the corresponding number
(b) (a) / (36)
(c) A Horn Bet is actually four bets in one
(d) Payout given the roll
(e) Net Payout considering other bets in a Horn Bet
(f) (b) * (e)

C-E Bet

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(profit)
2	1	2.78%	3	0.083
3	2	5.56%	3	0.167
11	2	5.56%	7	0.389
12	1	2.78%	3	0.083
Other	30	83.33%	-1	-0.833
Total		**100%**		**-0.1111**

(a) Number of ways to roll the corresponding number
(b) (a) / (36)
(c) Payout given the roll
(d) (b) * (c)

Any Seven

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(profit)
7	6	16.67%	4	0.667
Other	30	83.33%	-1	-0.833
Total		**100%**		**-0.1667**

(a) Number of ways to roll the corresponding number
(b) (a) / (36)
(c) Payout given the roll
(d) (b) * (c)

Any Craps

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(profit)
2	1	2.78%	7	0.194
3	2	5.56%	7	0.389
12	1	2.78%	7	0.194
Other	32	88.89%	-1	-0.889
Total		**100%**		**-0.1111**

(a) Number of ways to roll the corresponding number
(b) (a) / (36)
(c) Payout given the roll
(d) (b) * (c)

Hard 4

**Same procedure for Hard 10

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(profit)
[2, 2]	1	11.11%	7	0.778
[1, 3], [3, 1]	2	22.22%	-1	-0.222
7	6	66.67%	-1	-0.667
Total		**100%**		**-0.1111**

(a) Number of ways to roll the corresponding number

(b) (a) / (1+2+6)

(c) Payout given the roll

(d) (b) * (c)

Hard 6

**Same procedure for Hard 8

	(a)	(b)	(c)	(d)
Roll	# Ways	P(Roll)	Payout	E(profit)
[3, 3]	1	9.09%	9	0.818
[1, 5], [5, 1], [2, 4], [4, 2]	4	36.36%	-1	-0.364
7	6	54.55%	-1	-0.545
Total		**100%**		**-0.0909**

(a) Number of ways to roll the corresponding number

(b) (a) / (1+4+6)

(c) Payout given the roll

(d) (b) * (c)

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